Black hole catalysis of vacuum decay:

The semiclassical rate and importance of greybody factors

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w/ Andrey Shkerin

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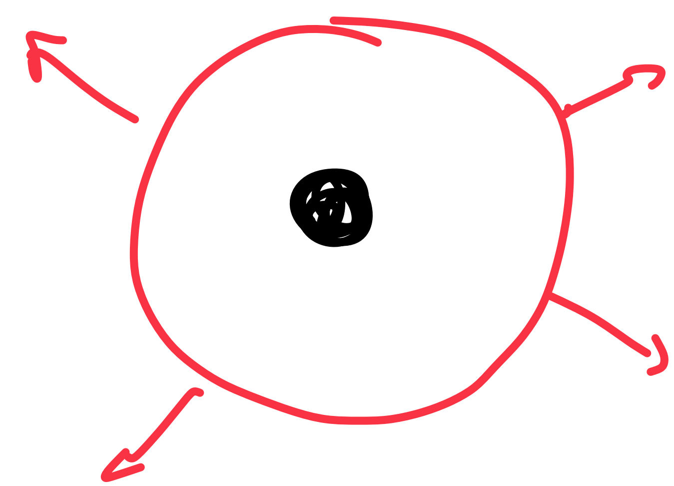

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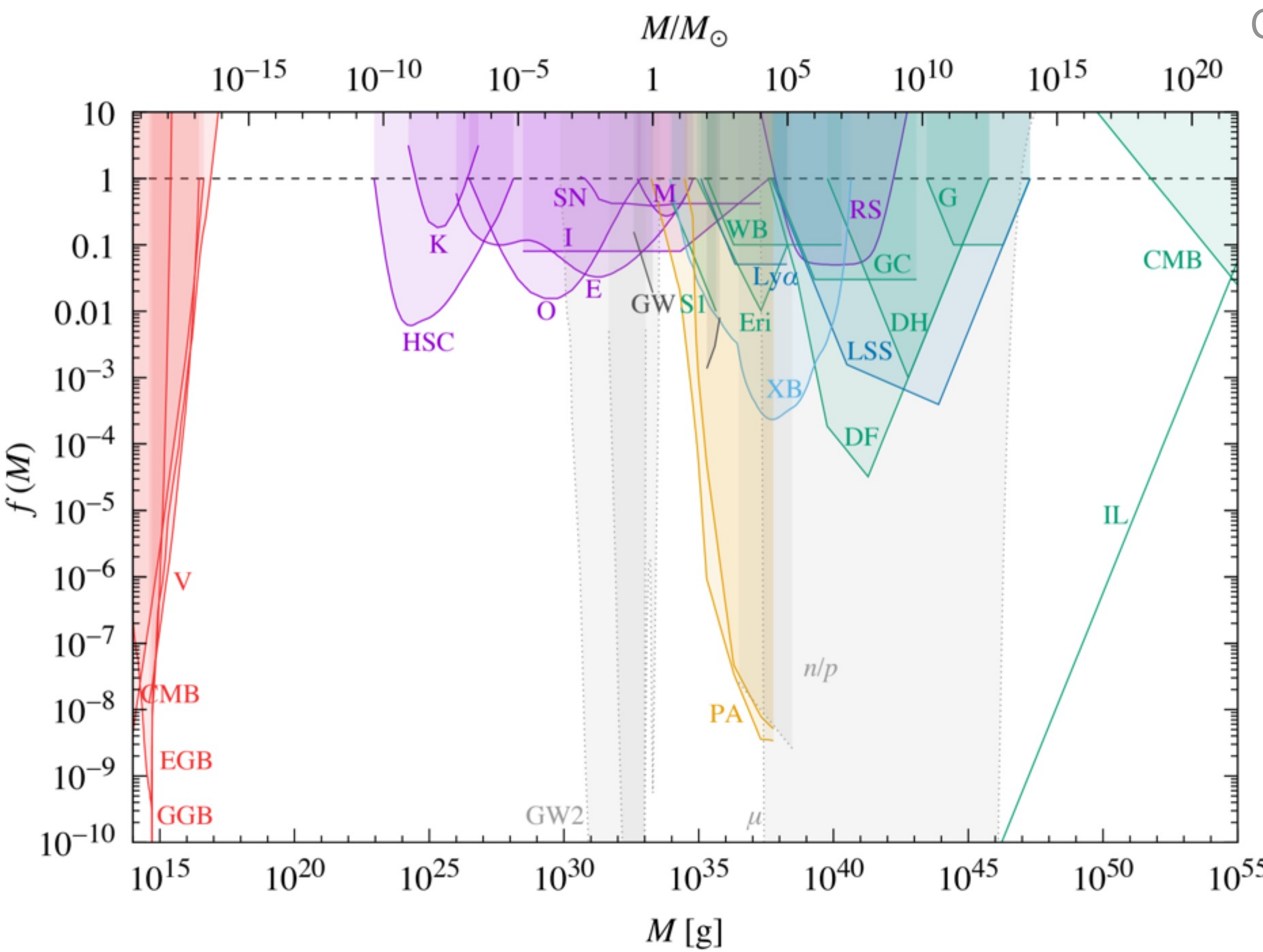
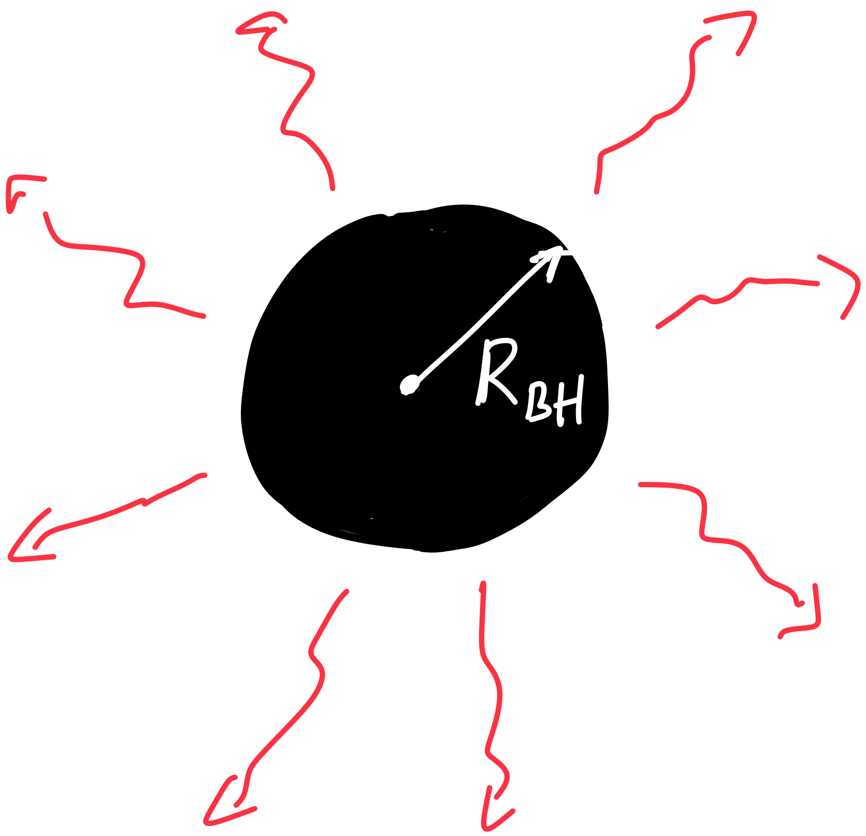
Primordial black holes in the universe

Carr et al. (2020)



lighter BHs evaporate by now

BH evaporation



$$T_{BH} \sim \frac{1}{R_{BH}} \sim \frac{M_{pl}^2}{M_{BH}}$$

$$\dot{M}_{BH} \sim -4\pi R_{BH}^2 \cdot T_{BH}^4$$

$$\Longrightarrow t_{BH} \sim \frac{M_{BH}^3}{M_{pl}^4} \sim 10^{64} \left(\frac{M}{M_\odot}\right)^3 \text{years}$$

$10^9 g < M_{BH} < 10^{17} g$: strong constraints from BBN, spectral distortions, etc.

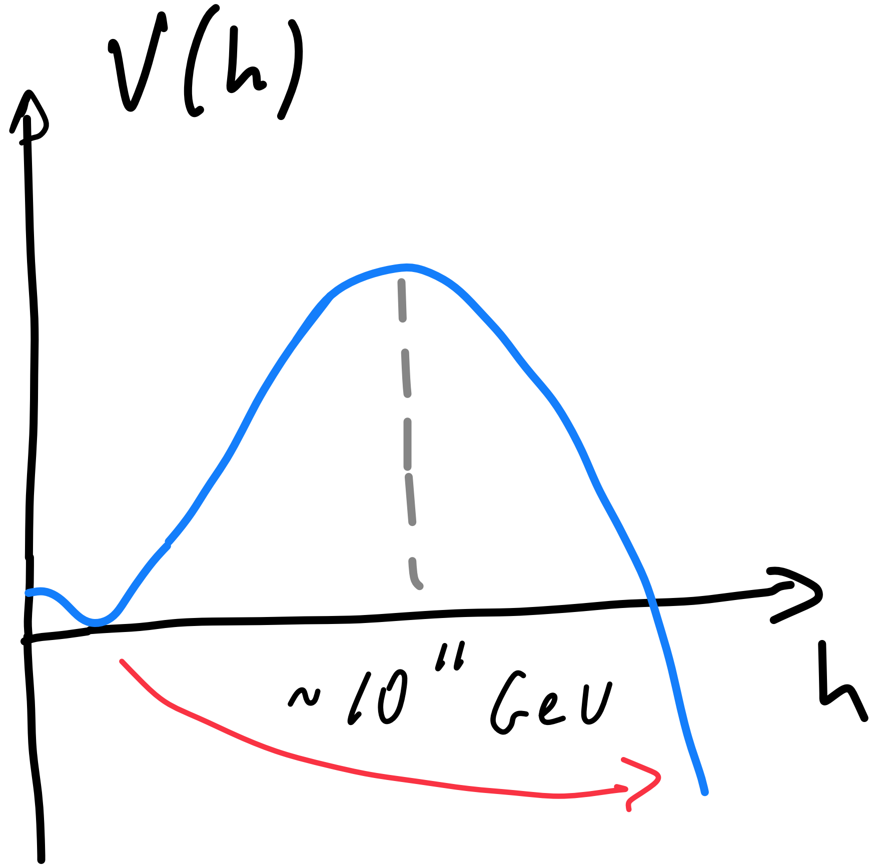
$M_{BH} < 10^9 g$: evaporate before BBN $\Longrightarrow$ no constraints

can do lots of interesting things: reheat universe, produce dark 9, generate baryon asymmetry, ... Garcia-Bellido et al (1996), Fujita et al (2014), Allahverdi et al (2017), Lennon et al (2017), Morrison et al (2018), Hooper et al (2019,2020), De Luca et al (2021)

Ultimate constraint on evaporating pBHs ?

Gregory, Moss, Withers (2014)
Burda, Gregory, Moss (2015)

Standard Model Higgs vacuum appears to be metastable



$$\Gamma_{decay} \sim h_{max}^4 \, e^{-B_0}$$

$B_0 = \frac{8\pi^2}{3|\alpha|}$ ← coupling constant

$$t_{decay} \gtrsim 10^{400} \text{ years}$$

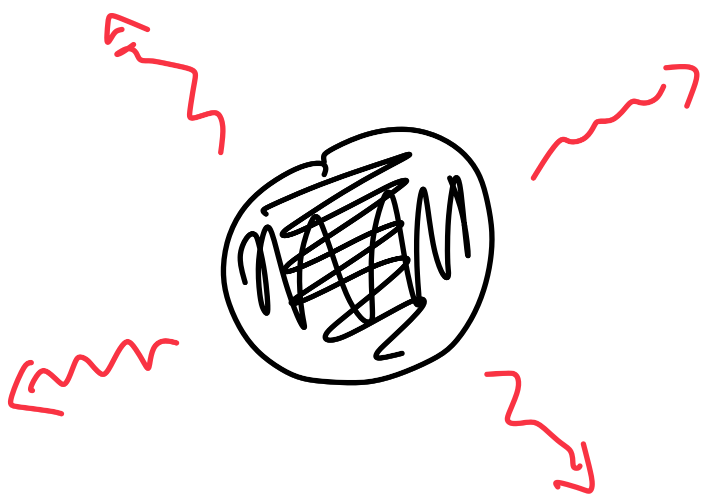
But the suppression decreases at high temperature

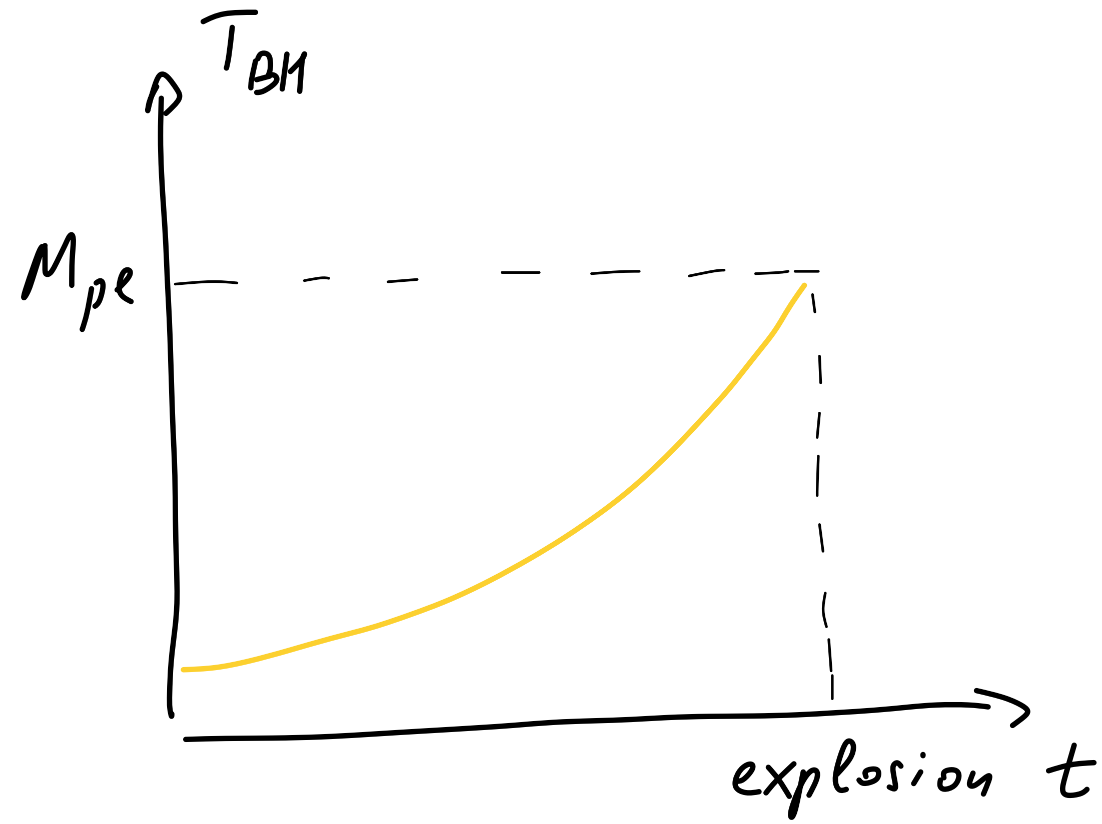
$$\Gamma_{decay} \sim h_{max}^4 \, e^{-\frac{E_{sph}}{T}}$$

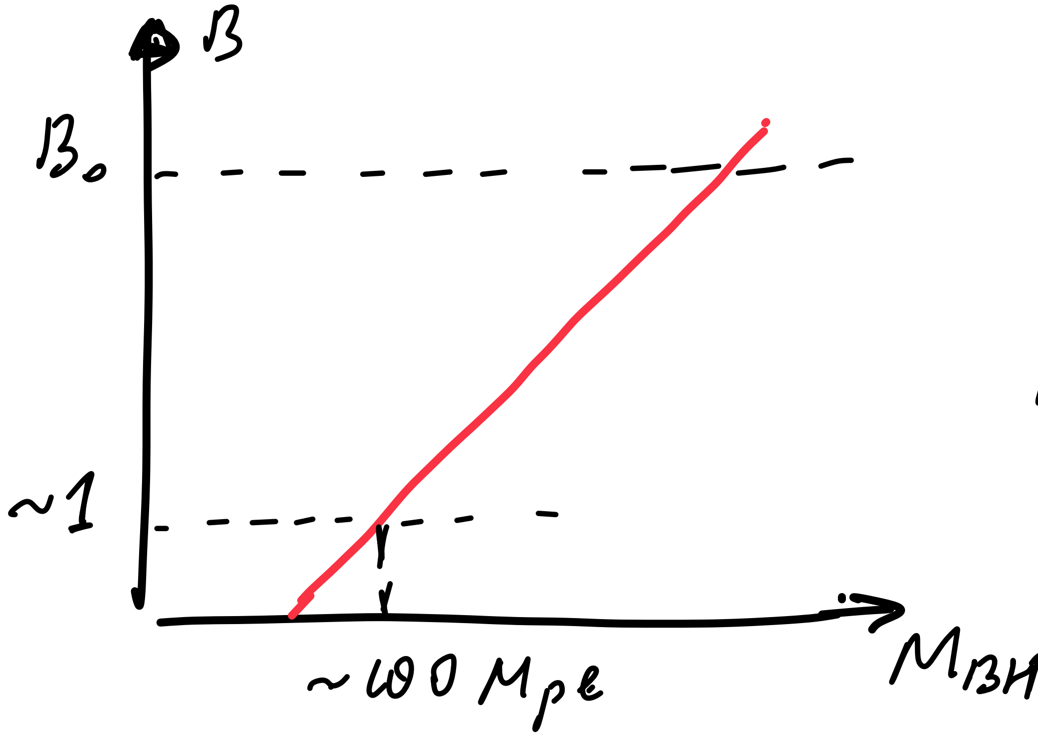
N.B. In SM E_{sph} gets large thermal corrections that may stabilize the electroweak vacuum in thermal bath

Ultimate constraint on evaporating pBHs ?

Combine vacuum metastability with evaporating BH





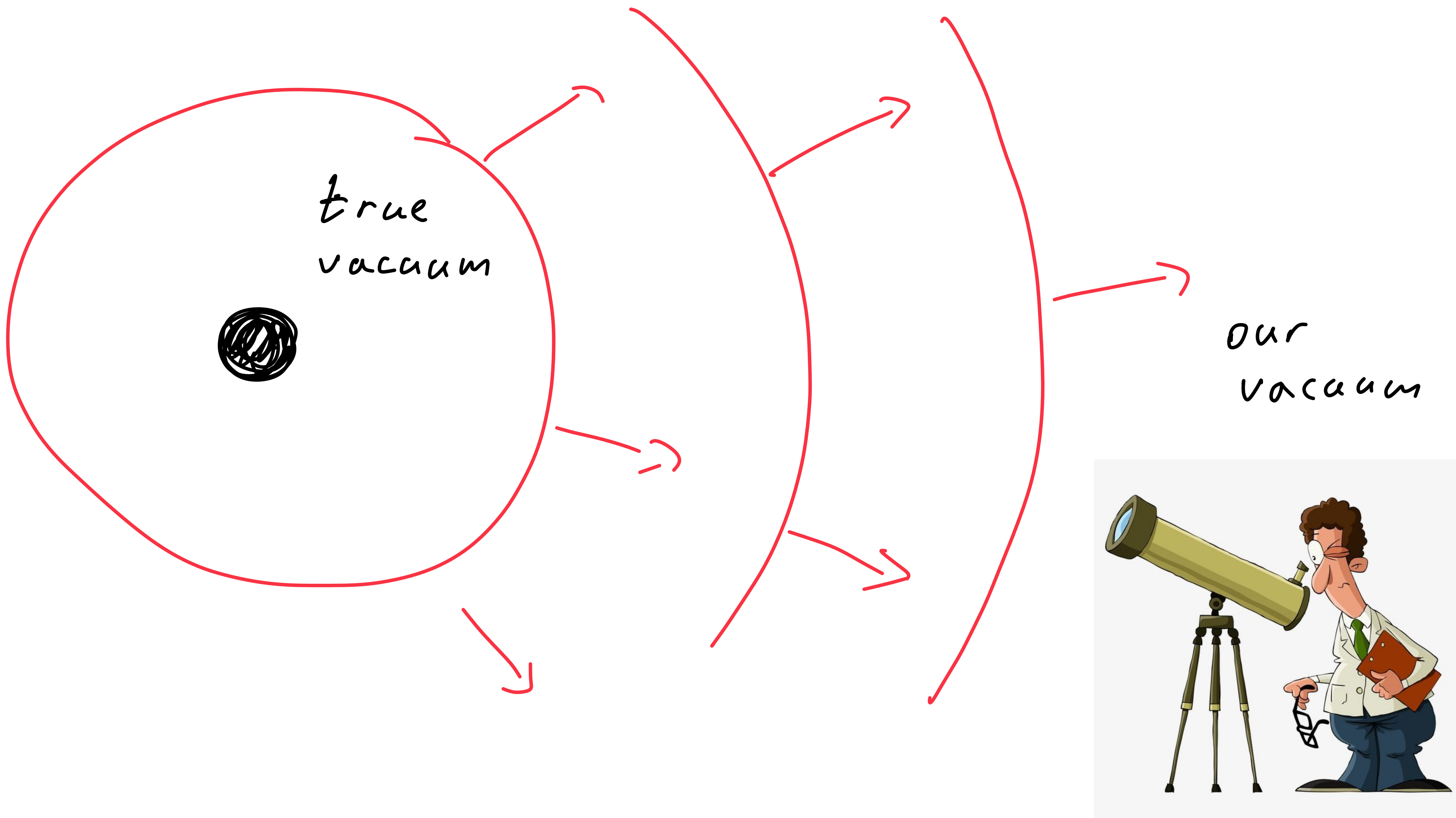


Burda et al (2015)

$$\Gamma \sim h_{max}\, e^{-B}$$

Ultimate constraint on evaporating pBHs ?

Even a single evaporating BH would trigger decay of the electroweak vacuum!

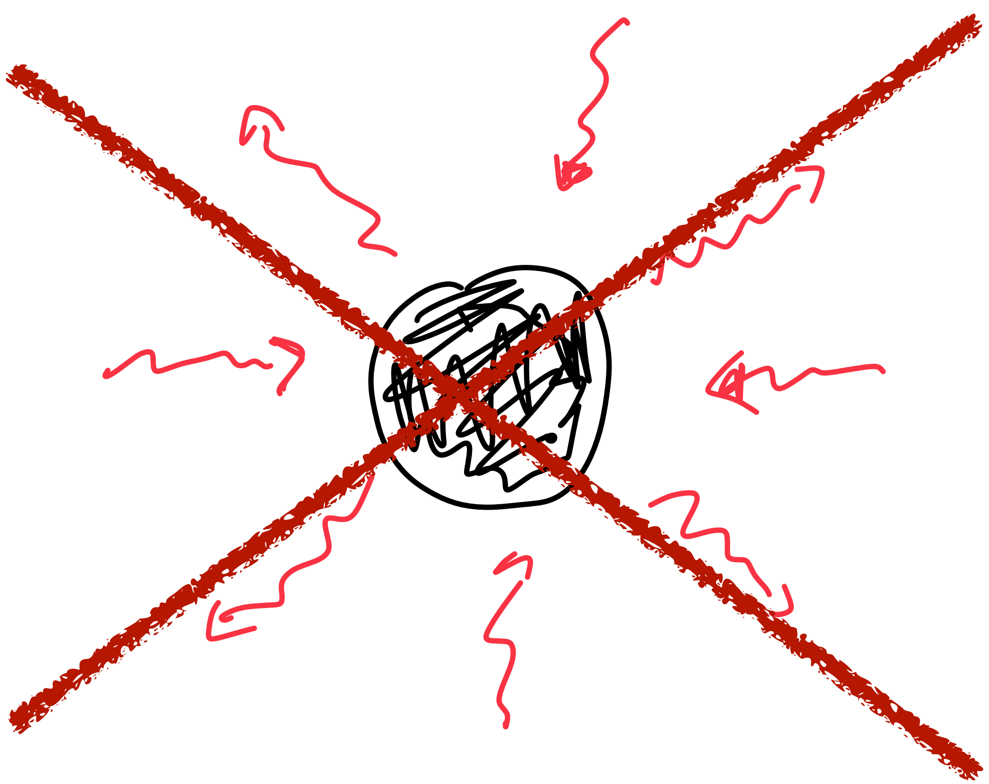


Not a single evaporating BH in our past!

Caveat

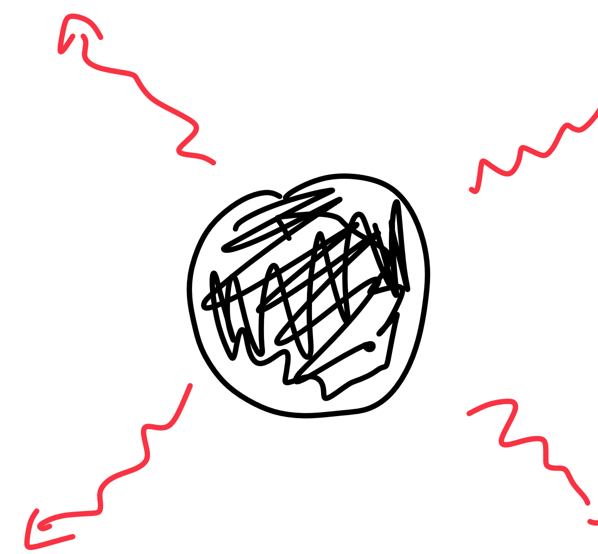
the calculation uses Euclidean instantons

suitable for thermal equilibrium



Harle-Hawking vacuum

but realistic BH is not in thermal equilibrium with its environement

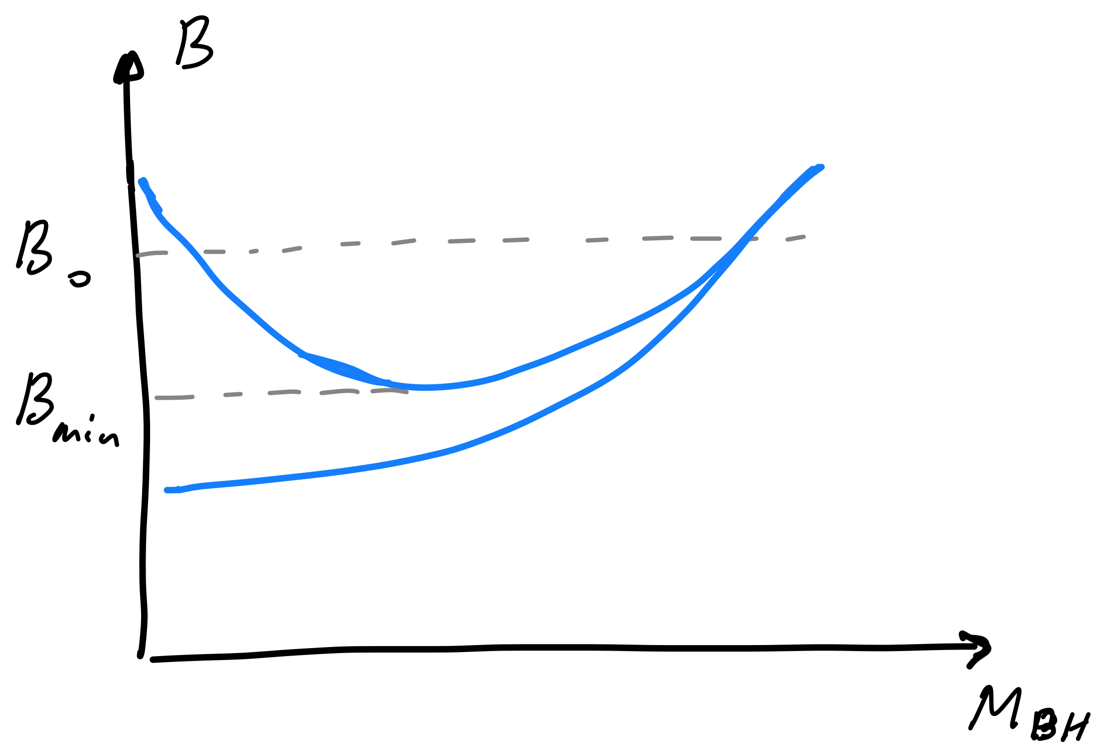


Unruh vacuum

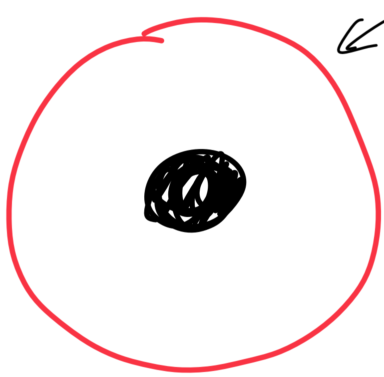
Realistic black hole

Gorbunov, Levkov, Panin (2017)

Expected:



Argument 1



critical bubble

$$R_{cb} \gg R_{BH}$$

$$n_{soft}\big|_{R_{BH}} \sim \frac{T_{BH}}{\omega} \gg 1$$

$$n_{soft}\big|_{R_{cb}} \sim n_{soft}\big|_{R_{BH}} \left(\frac{R_{BH}}{R_{cb}}\right)^2$$

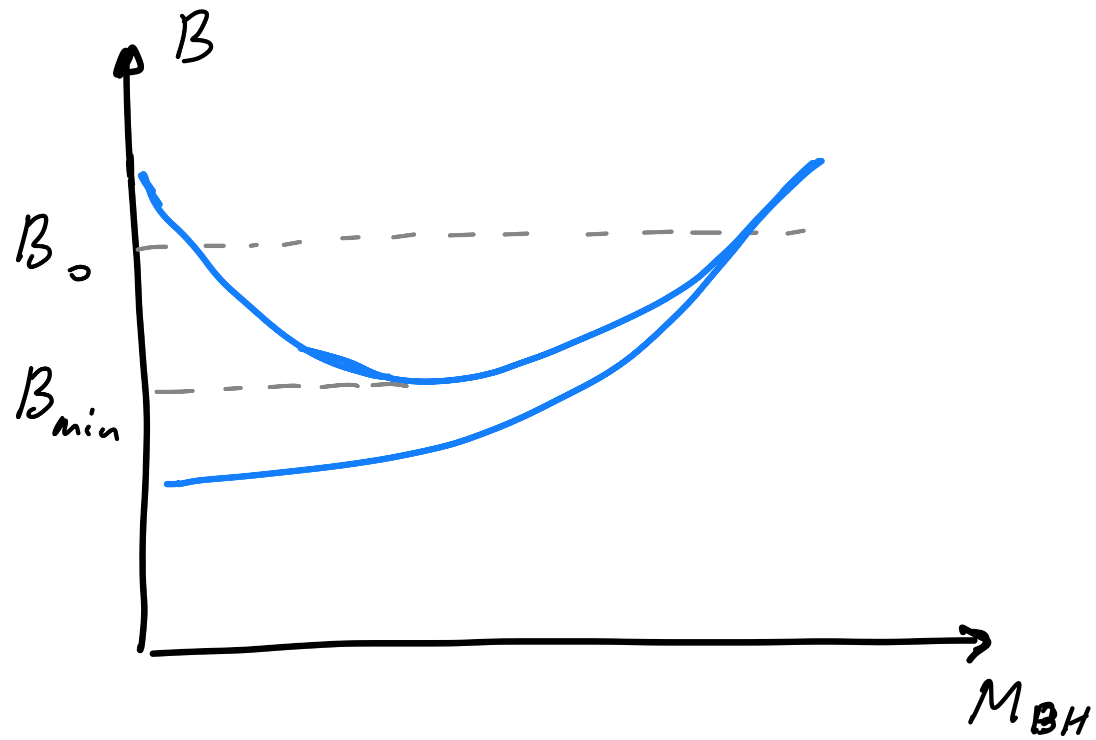
$$\sim \frac{1}{(\omega R_{cb})} \left(\frac{R_{BH}}{R_{cb}}\right) \sim \frac{R_{BH}}{R_{cb}} \ll 1$$

not enough quanta to make a classical critical bubble

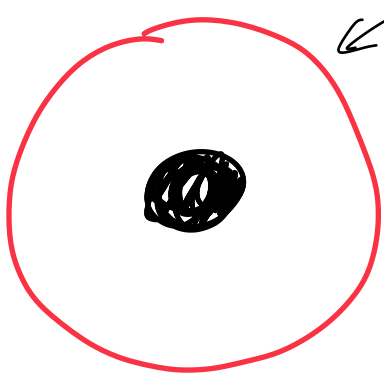
Realistic black hole

Gorbunov, Levkov, Panin (2017)

Expected:



Argument 2



critical bubble

$R_{cb} \gg R_{BH}$

n_{soft} further suppressed by greybody factors

$$P_{\substack{soft \\ emission}} \sim (\omega R_{BH})^2 \sim \left(\frac{R_{BH}}{R_{cb}}\right)^2$$

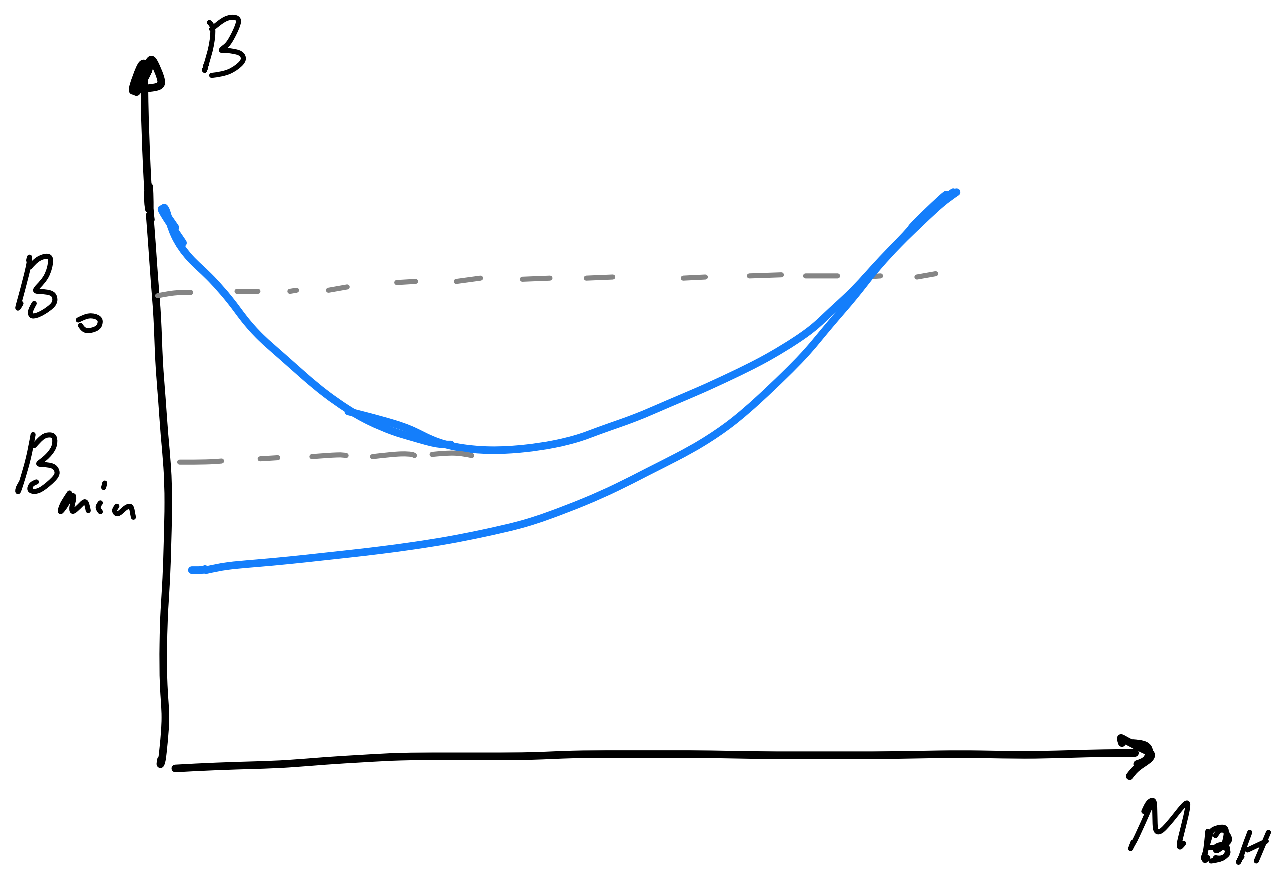
to make a critical bubble we need

$$N_{soft} \sim \frac{1}{|\alpha|} \Rightarrow P_{cb} \sim \left(\frac{R_{BH}}{R_{cb}}\right)^{\frac{2}{|\alpha|}}$$

exponentially small

Realistic black hole

Expected:



We want to know how to calculate β_{min} :

the Universe is big and even exponentially suppressed rates can give non-trivial constraints on pBH models

Other reasons to study BH catalysis

- non-perturbative non-local process

$\Longrightarrow$ teaches us smth about QFT in curved geommetry

- semiclassical description can potentially take into account gravity

$\Longrightarrow$ teaches us smith about semiclassical quantum gravity
(cf. Coleman-De Lucia / Hawking-Moss instanton)

- the BH mass can change in the process *(increase / decrease?)*

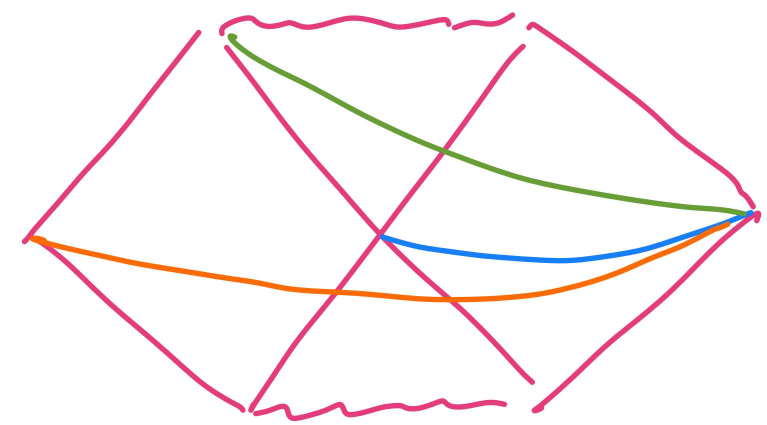
$\Longrightarrow$ teaches us smth about BH entropy / information paradox

(Not so) technical puzzles

- ***where does bounce live?*** cannot be Euclidean time

- ***what quantization surface we shall choose for the field?***
  does tunneling depend on what's going on inside BH?

complete BH spacetime includes interior
and another asymptotic region...



- ***what are the conditions ensuring that bounce starts from Unruh vacuum?***

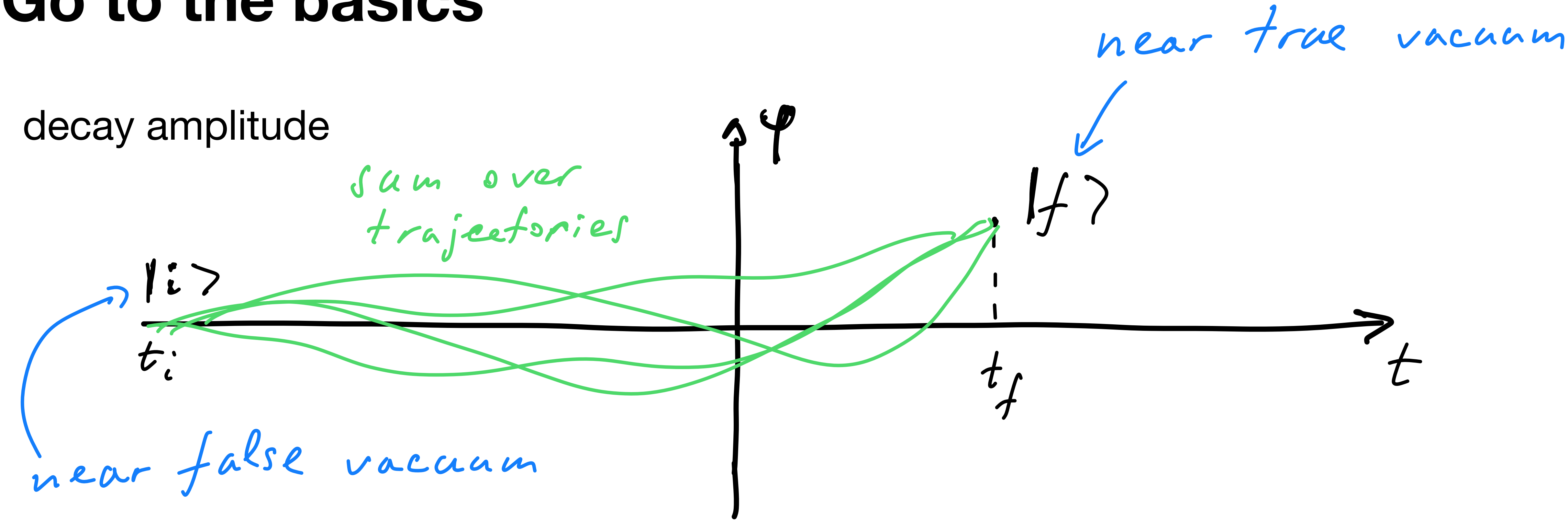
$\Rightarrow$ **Address these questions in external BH metric**

N.B. change of BH mass is irrelevant as long as $R_{bubble} \ll t_{evap}$

Γ_{decay} can be bigger or smaller than t_{evap}^{-1}

Go to the basics

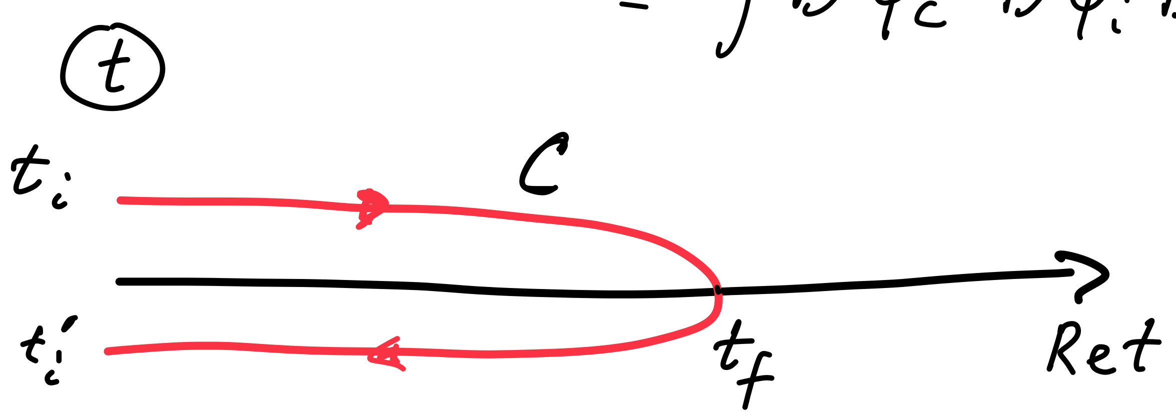
decay amplitude



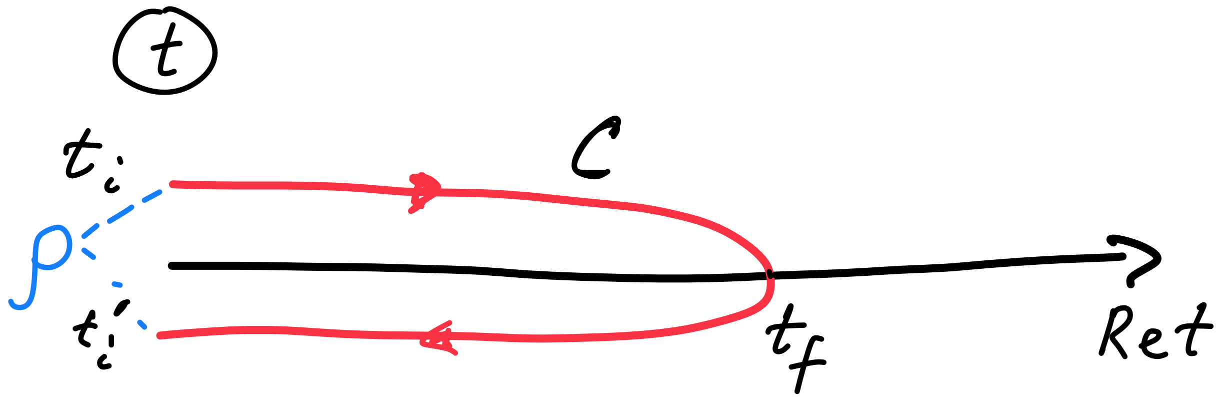
decay probability

$$P_{decay} = \sum_f \langle i | \underbrace{f\rangle\langle f}_{\text{projector on true vacuum}} | i \rangle$$

$$= \int D\varphi_c \, D\varphi_i \, D\varphi_i' \, \langle i | \varphi_i', t_i' \rangle \, e^{iS[\varphi_c]} \langle \varphi_i, t_i | i \rangle$$



for mixed states



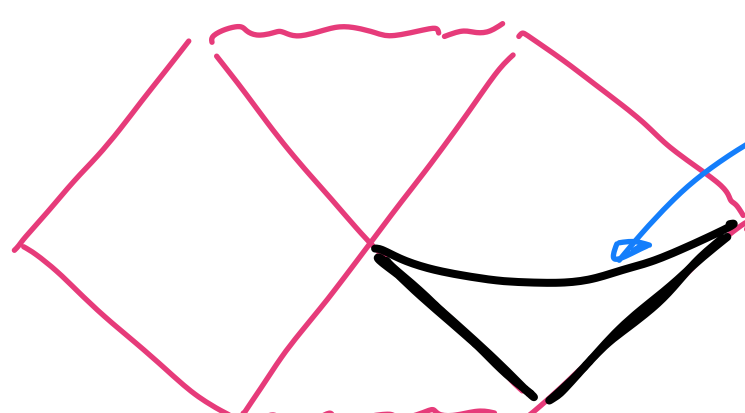
density matrix

$$P_{decay} = \int D\varphi_c \, D\varphi_i \, D\varphi_i' \, e^{iS[\varphi_c]} \underbrace{\langle \varphi_i \, t_i | \rho | \varphi_i' \, t_i' \rangle}$$

bounce is the saddle-point,
lives on the in-in contour *C*

sets the boundary conditions at the ends of the contour: same as for the time-ordered Green's function

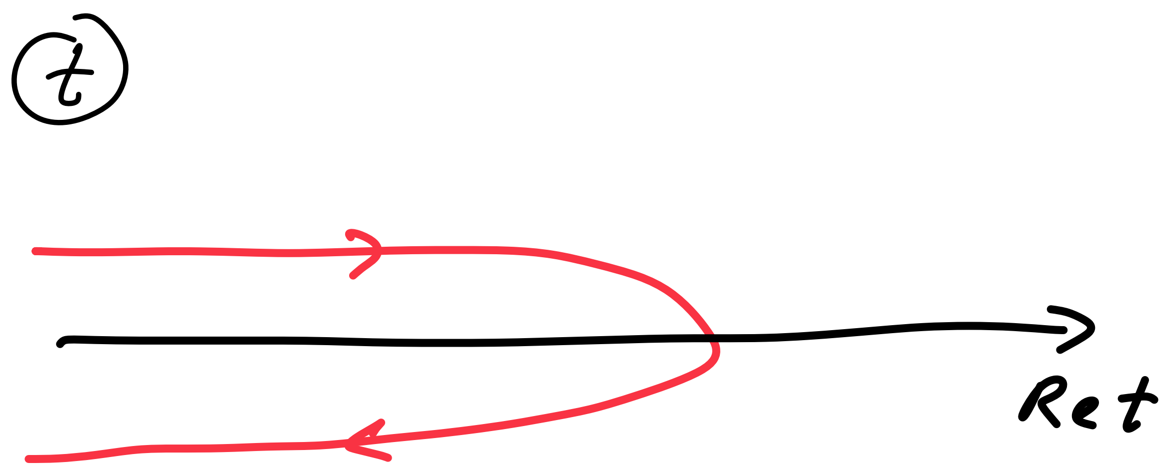
We can stay outside BH: just treat H-H and Unruh vacua as mixed states



quantization surface

Summary I

We need to solve <u>classical</u> field eqs. on a contour in complex time plane

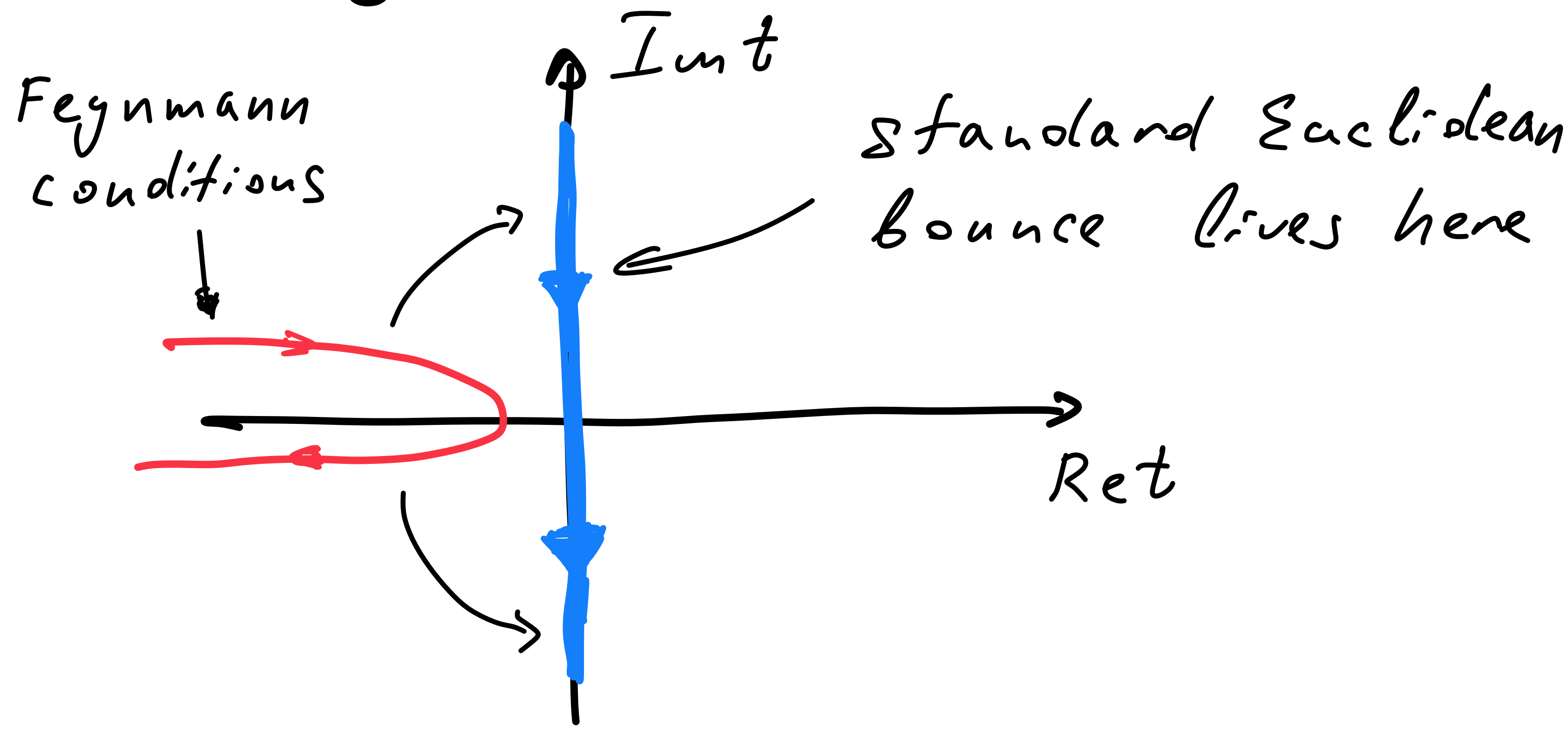


with BC's fixed by the causal Green's function

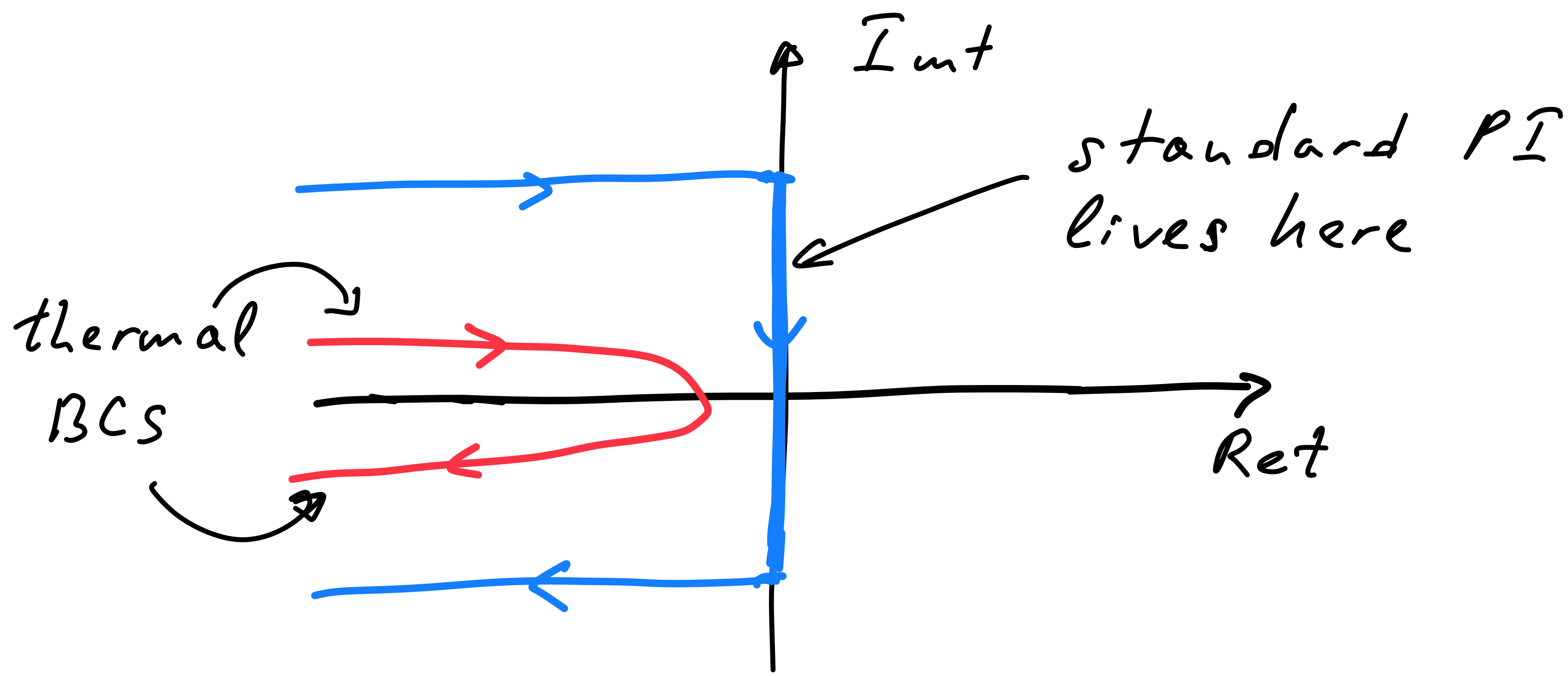
$$\Gamma \sim e^{-B}, \quad B = -i\, S[\varphi_{bounce}]$$

Contact with prior knowledge

vacuum bounce



periodic instantons



N.B. Can also treat formation of sphaleron (over-barrier jumps) at high temperature

In general, no useful Euclidean picture

$\Longrightarrow$ hard numerical problem

seems the case for Unruh vacuum

$\Longrightarrow$ **go to (1+1) dimensions!**

"A man grows stale if he works all the time on insoluble problems, and a trip to the beautiful world of one dimension will refresh his imagination better than a dose of LSD."

Freeman Dyson

Toy model 1

dilaton black hole

$$ds^2 = -\Omega(r)\,dt^2 + \frac{dr^2}{\Omega(r)}, \qquad \Phi = -\lambda r$$

$$\Omega(r) = 1 - \frac{M}{2\lambda} e^{-2\lambda r} \Rightarrow \text{horizon at } r_{BH} = \frac{1}{2\lambda} \ln \frac{M}{2\lambda}$$

temperature: $T_{BH} = \frac{\lambda}{2\pi}$

N.B. does not depend on BH mass

in tortoise coordinates:

$$ds^2 = \Omega(x)\left(-dt^2 + dx^2\right)$$

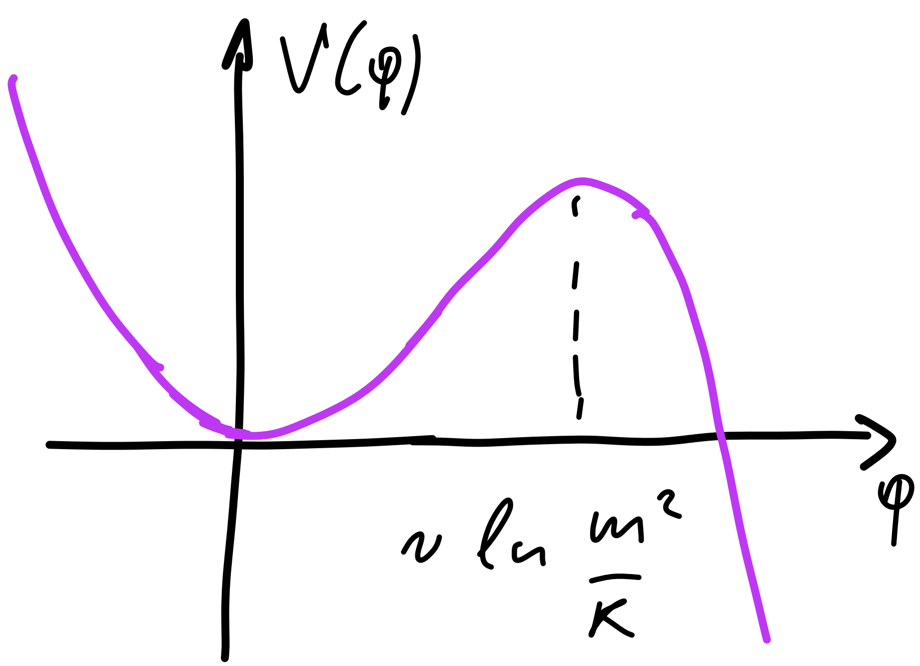
$$\Omega(x) = \frac{1}{1 + e^{-2\lambda x}}$$

Toy model 1

inverted Liouville potential with a mass

$$S = \frac{1}{\alpha^2}\int d^2x\,\sqrt{-g}\,\left(-\frac{1}{2}(\nabla\varphi)^2 - V(\varphi)\right) \qquad \boxed{\alpha \ll 1}$$

$$V(\varphi) = \frac{m^2\varphi^2}{2} - 2\kappa\left(e^{\varphi} - 1\right)$$



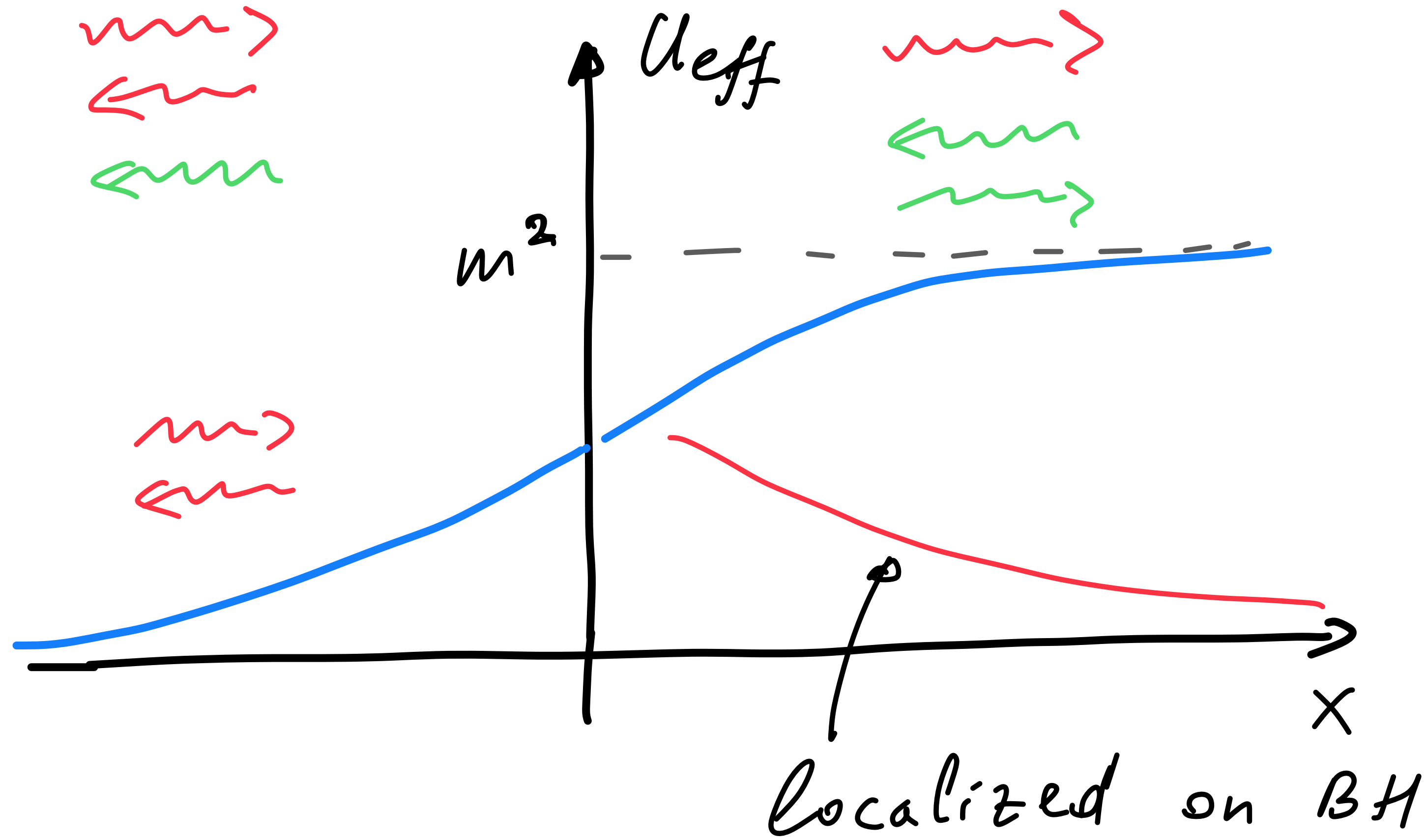
assume : $\boxed{\ln\frac{m}{\sqrt{\kappa}} \equiv L \gg 1}$

instrumental to
find analytic
bounce solutions

N.B. We do not add non-minimal coupling to gravity

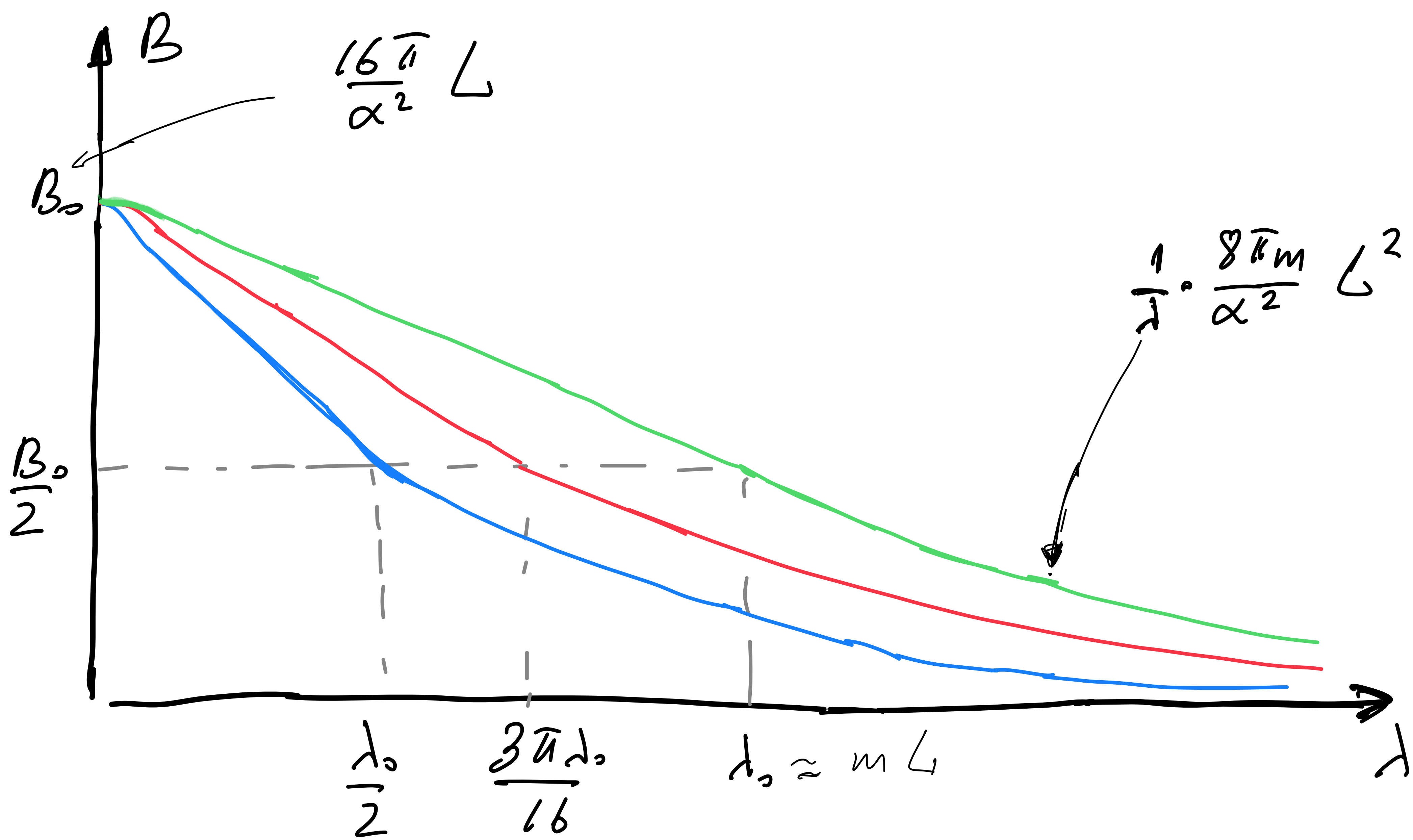
Effective potential for linear modes

$$-f'' + m^2\Omega f = \omega^2 f$$



Green's functions are dominated by soft modes with $\omega \sim m \ll d$

Results 1



— thermal rate in flat space

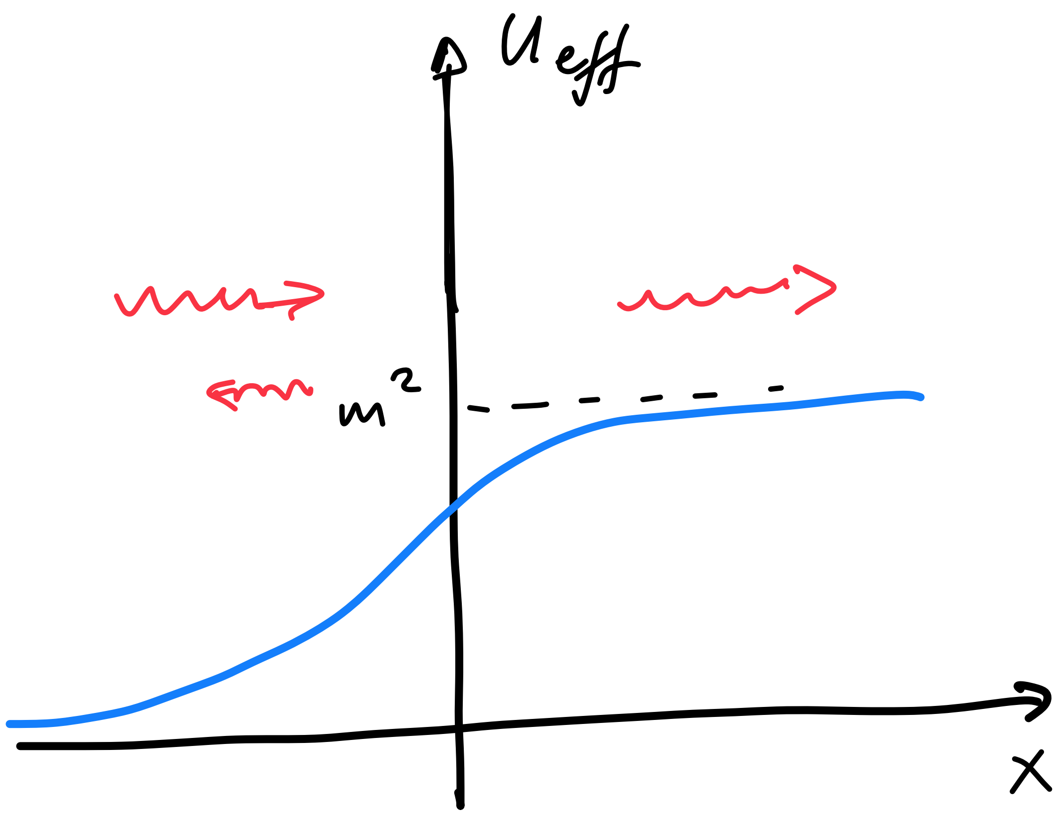
— decay rate of Hartle-Hawking vacuum

— decay rate of Unruh vacuum

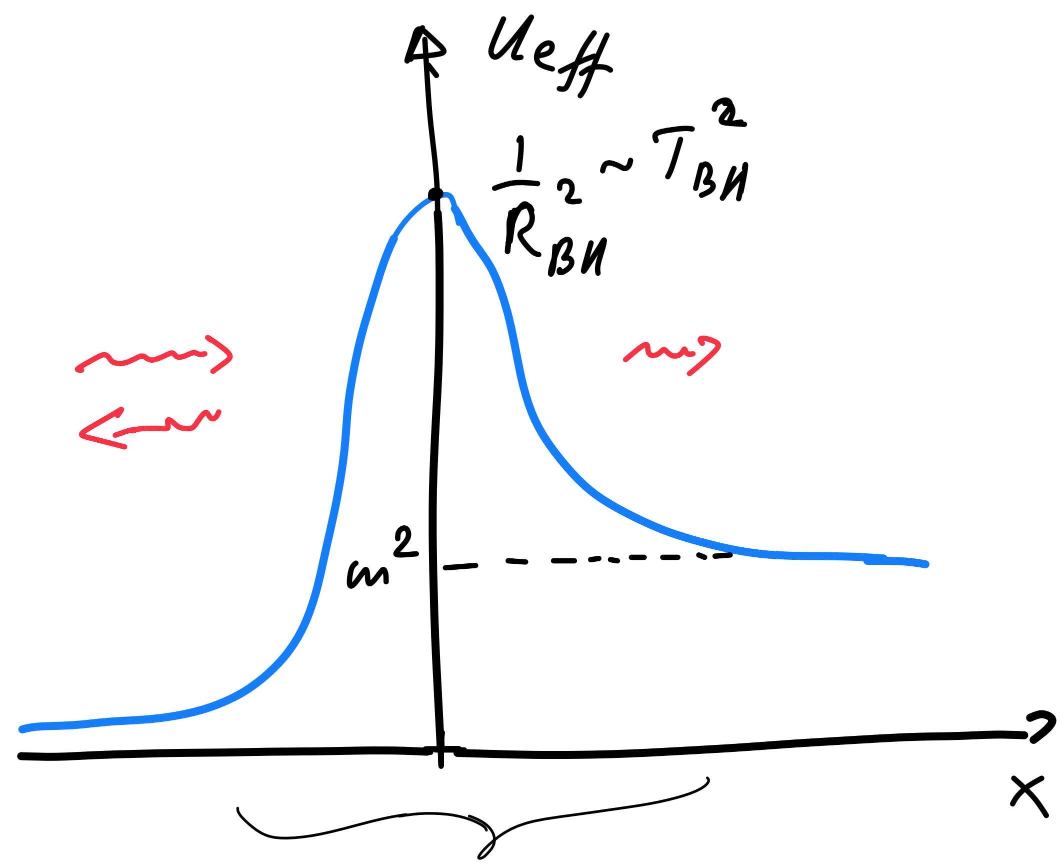
Have we missed anything?

Greybody factors!

2d



4d

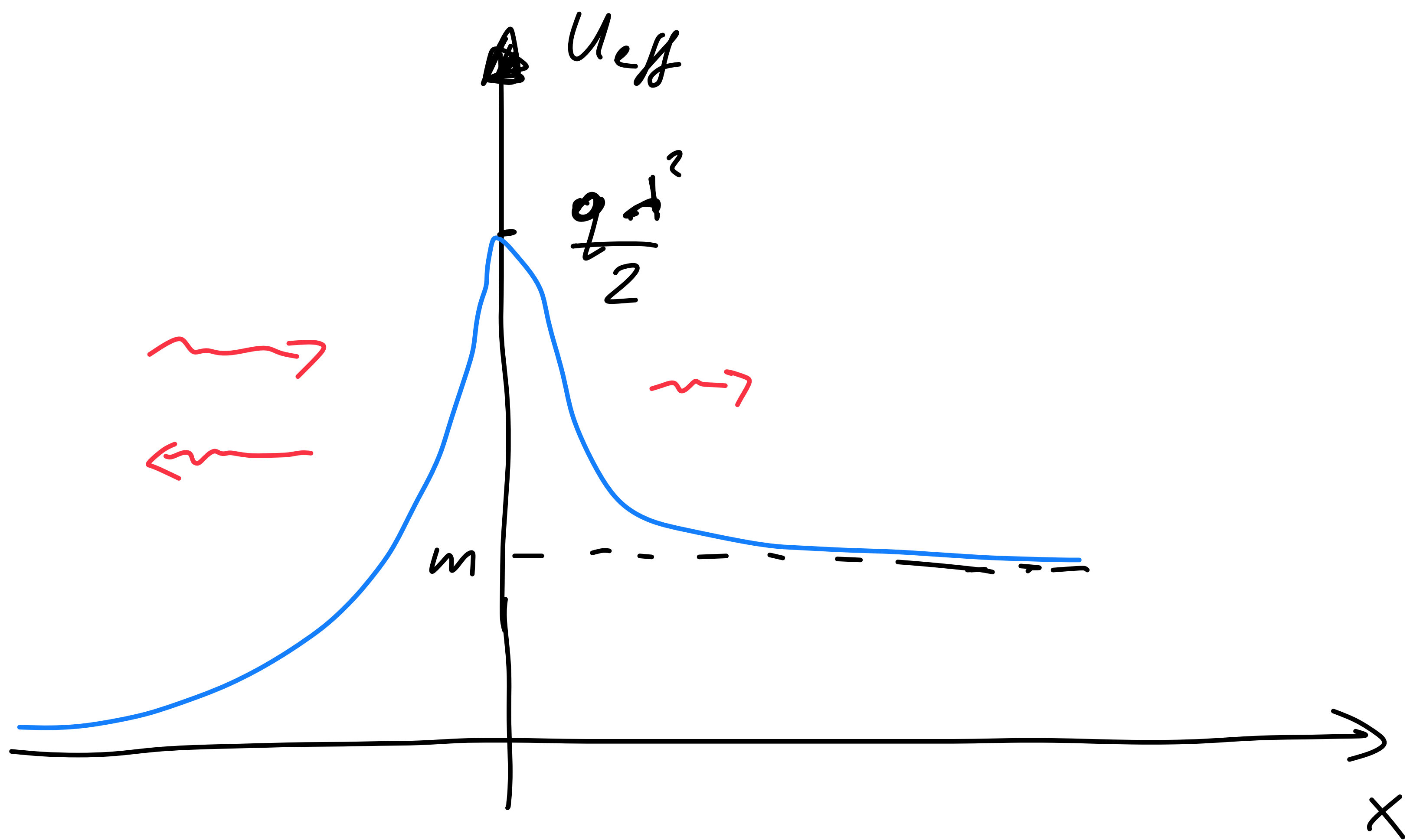


centrifugal barrier

Toy model 2

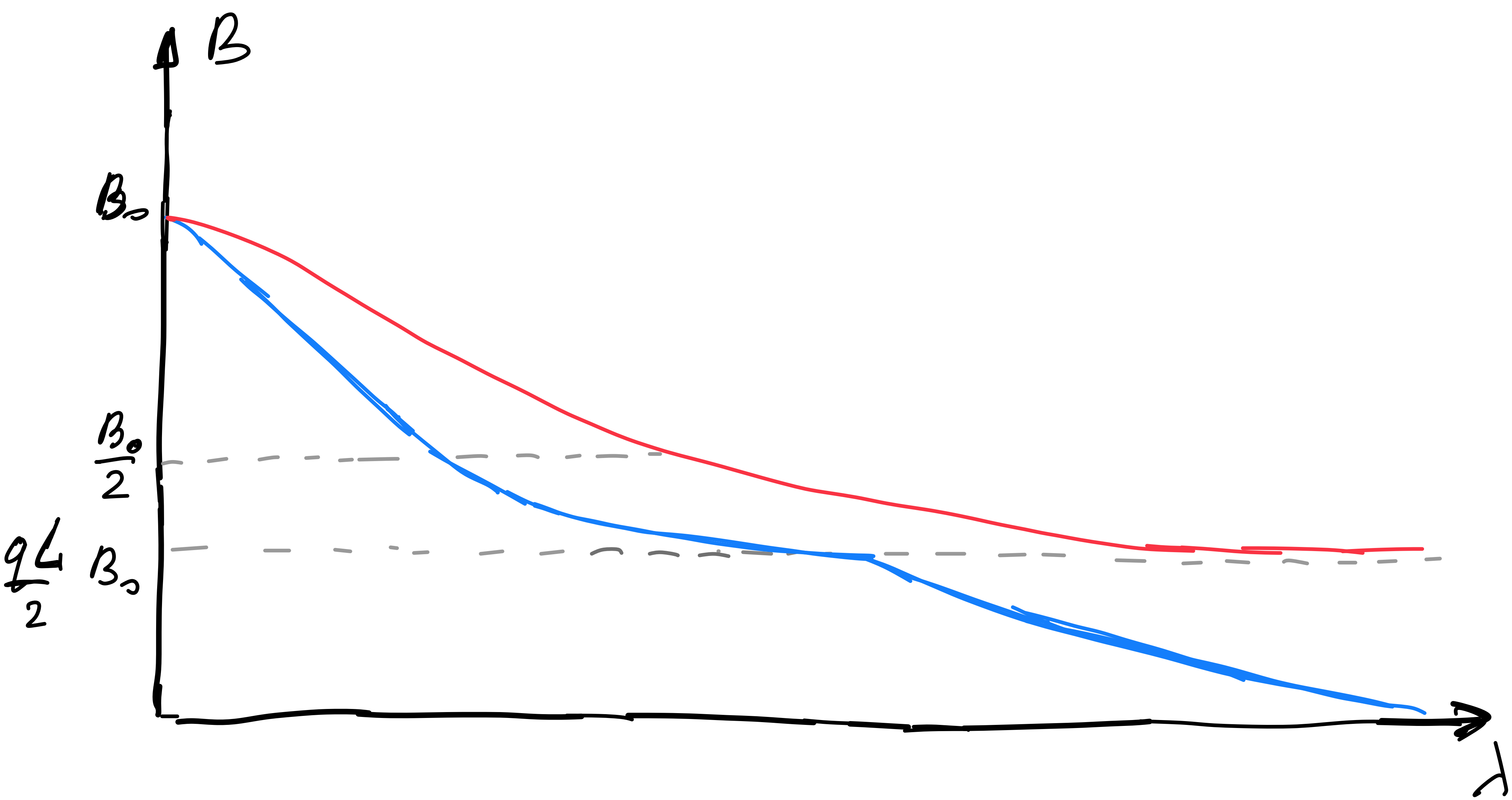
Can we model the potential barrier in 2d?

Yes: add interaction between the scalar and the dilaton $\quad Q\, e^{2\Phi} \varphi^2$



If $\quad q \ll 1$, the model is again analytically solvable

Results 2



— decay rate of Hartle-Hawking vacuum

— decay rate of Unruh vacuum

Summary I I

- The toy mode allowed us to explicitly see the difference for decay of different vacua in BH background
- Unruh decay is more suppressed than Hartle-Hawking
- When greybody factors are accounted for, Unruh suppression goes to a non-zero constant at high T_{BH} The rate remains exponentially suppressed, but larger than in empty space

Outlook

- Realistic calculation for BH in 4d
- More general setup: e.g. BH in thermal plasma with $T_{plasma} \neq T_{BH}$
- Include back-reaction on the metric